Explicit formulas for Glaeser-Whitney types extensions using sup and inf-convolutions

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We are interested in the following Glaeser-Whitney type problem of extension of 1-Taylor fields. In a Hilbert space H, let S be any subspace and $(a(s), v(s))_{s\in S}$ be a 1-Taylor field (i.e., a family of affine functions $a(s) + \langle v(s), \cdot - s\rangle$ for $s \in S$). The goal is to find a function $F : H \to \mathbb{R}$, which is of class $C^{1,1}(H)$ and satisfies $F(x) = a(x)$ and $\nabla F(x) = v(x)$ for all $x \in S$. This problem was solved by Whitney (1934) and Glaeser (1958) in finite dimension and by Wells (1973) and Le Gruyer (2009) in Hilbert spaces. The proofs are complicated and use Whitney covering techniques or Zorn Lemma. Instead, we give a simple proof using sup and inf-convolutions techniques. This proof gives naturally explicit formulas for the extension. This is a joint work with Aris Daniilidis (Santiago) and Mounir Haddou, Erwan Le Gruyer (Rennes).