Convergence of a threshold-type algorithm to curvature-dependent motions

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Let $\{\Gamma(t)\}_{t\in[0,T)}$ be a family of compact hypersurfaces in $\mathbb{R}^N$. We call it a curvature-dependent motion (CDM for short) if $\Gamma(t)$ moves by the following equation

$$V = \kappa + \langle \mathbf{b}, \mathbf{n} \rangle + g \quad \text{on } \Gamma(t),\ t \in (0, T).$$

Here $T > 0$, $\mathbf{n} = \mathbf{n}(t, x)$ is the inner unit normal vector field on $\Gamma(t)$, $V = V(t, x)$ is the velocity of $\Gamma(t)$ in the direction of $\mathbf{n}$, $\kappa = \kappa(t, x)$ is the ($(N-1)$-times) mean curvature of $\Gamma(t)$, $\mathbf{b} = \mathbf{b}(t, x)$ denotes a given vector field, $g = g(t, x)$ is a forcing term and $\langle \cdot, \cdot \rangle$ denotes the inner product in $\mathbb{R}^N$. As well known, the mean curvature flow is the case of $\mathbf{b} \equiv \mathbf{0}$ and $g \equiv 0$. The CDM arises in various fields such as two-phase problems, image processing, and so on.

In this talk we introduce a threshold-type algorithm for the above motion. Roughly speaking, we iteratively use the solutions of the initial-val;ue problem fot the linear parabolic equation. The main purpose of this talk is to present the convergence to the generalized CDM and its optimal rate to the smooth and compact CDM.

This is based on my joint work with Professor Masato Kimura (Kanazawa University) and Mr. Takahiro Izumi (Yasuna Machine Designing).