Some generalizations of the vanishing discount problem and the comparison principle

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A recent development on Hamilton-Jacobi equations is concerned with the question if the solutions $u^\lambda(x)$ of the discounted problem $\lambda u + G(x, Du(x)) = 0$ converges to a solution of $G(x, Du(x)) = 0$. A work by Davini at al. gave a fairly complete answer to this vanishing discount problem. I discuss some convergence results for the Hamilton-Jacobi equation $H^\lambda(x, Du(x), u(x)) = 0$ in M, where M is a compact, connected, smooth manifold without boundary and λ is a positive constant. We assume that the function $H^\lambda(x, p, u)$ is convex and coercive in p and strictly increasing in u and that, in an appropriate sense, $H^\lambda(x, p, u)$ is approximately equal to $\lambda u + G(x, p)$ as λ tends to zero. This is based on joint work with Q. Chen, W. Cheng, and K. Zhao.